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Motion estimation and modeling of the environment for underwater vehicles

ROBERTO CRISTI†, MASSIMO CACCIA† and GIAMMARCO VERUGGIO‡

The problem of localizing an underwater vehicle in a structured environment using sonar returns and measured acceleration is addressed. In particular the environment is modeled as a sequence of segments with constant orientation. The combined estimation of the vehicle motion and the orientation of the segments is obtained using an extended Kalman filter based on a suitable state-space model. Several experiments in the test tank show the effectiveness of this technique.

1. Introduction

Underwater missions require the vehicle to position itself relative to the environment in which it is operating. This is the case of a vehicle performing surveying operations around an oil production facility, or in the important problem of self-docking and rendezvous operations.

Sensing the environment without tactile modality can be performed by vision, sonar or a combination of these two. In certain cases a vision-based approach might not be attractive owing to murky water, poor visibility conditions and the need for lighting.

Varying degrees of precision are needed relative to the particular task. For example, position referencing for oceanographic sampling may allow errors of the order of several meters over ranges of a few kilometers. Referencing with standard long-baseline techniques over short ranges, however, using high-frequency transponders may allow precision to within centimeter accuracy. Also using on-board high-frequency sonars can provide the ability to acquire shape and orientation of structures such as an underwater dock, as opposed to absolute position alone. Vision, of course, at short ranges provides rich feature characterization.

The use of sonar sensors for both land vehicles and underwater vehicles has been presented in a number of research works. Most of the work done at the present time is two dimensional as an extension of the research done for land vehicles (Moran 1993).

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The use of sonar sensors for both land vehicles and underwater vehicles has been presented in a number of research works. Most of the work done at the present time is two dimensional as an extension of the research done for land vehicles (Moran 1993).

In this research we address a two-dimensional navigation problem of an underwater vehicle localization using sonar returns from known features. For the time being, we assume the objects to have sufficient reflectivity so that they can be distinguished from other irrelevant features without additional signal processing.

A typical mission is described by a vehicle working in an area where features are present, with its tasks being dependent on its exact localization with respect to the environment. This can be the case of a vehicle operating next to platforms, piers or moored ships.

In this paper we address the problem of a vehicle navigating in an environment which could be both unknown or partially known. The assumption that we make on the obstacles is that their reflective surfaces are piecewise modeled by segments having constant orientation in two dimensions. The goal of the algorithm that we develop is then to provide the best estimate of the vehicle's motion and of the segments of the environment, in terms of location and orientation. We assume that the vehicle's initial location and velocity are known, and we measure its acceleration. This can also be combined with other measurements such as an acoustic tracker for an estimate of its position, or a Doppler sonar for velocity measurement.

The estimated environment is then memorized in terms of a potential function to be used to correct the vehicle's estimated trajectory, along the lines described by Cristi *et al.* (1995). The potential function provides a mean for error correction using look-up tables, and it is constructed using the estimated location of the reflecting surfaces and their orientation. A scheme for localizing the vehicle has been presented by Cristi *et al.* (1995), based on an adaptive filter.

The potential function approach seems to be particularly attractive, in the sense that not only does it not rely on a geometric model (lines, circles, etc.), but also it can be at the basis of the determination of the controller to carry on the mission. To this end, several papers have recently appeared in the literature (Rimon and Kordistchek 1992, Guldner and Utkin 1995) which address the problem of a joint design of a path planner and corresponding controller on the basis of a suitable potential function. Goal of future research will be the joint combination of vehicle localization and control under the same framework.

2. World and vehicle modeling

In this section we address the problem of modeling the vehicle motion and the environment in which it operates. The goal is to determine a framework which allows one to combine the information from the inertial navigation system (gyros and accelerometers), with the onboard sensors (such as sonar and, possibly, vision) for the following purposes:

(*a*) to localize the vehicle with respect to known landmarks;

(*b*) locate obstacles with respect to known vehicle location;

(*c*) to attempt to localize the vehicle and the surroundings with respect to known initial conditions (position and velocity).

All three tasks above are based on the same dynamic models for the vehicle and the environment. Particularly challenging is (*c*), where we envisage a mission where the vehicle *explores* the surroundings guided by its own inertial navigation equipment (possibly dead reckoning from either acceleration or velocity measurements) while it maps the environment. Then it uses this acquired information to guide itself within the environment. The success of this task would lead to an operation where—ideally at least—the vehicle can operate without an active short-baseline system, guided by passive reflecting surfaces (*brilliant rocks*).

In all these cases the dynamics of the vehicle moving in a three-dimensional environment is represented by a state vector determined by its *position* p with respect to a reference frame fixed with the environment, a *velocity* and *orientation* vector z and a command vector u involving the commands given to the actuators of the vehicle.

The dynamics can then be expressed as a combination of kinematics and dynamic equations of the form

$$\left.\begin{aligned} \dot{p} &= f_1(z), \\ \dot{z} &= f_2(z, u), \end{aligned}\right\} \tag{1}$$

with the mappings f_1 and f_2 representing kinematic and dynamic relations respectively.

An alternative representation of the vehicle's dynamic is purely kinematic, where we assume to measure its linear acceleration. In this case the dynamic equation of the vehicle becomes

$$\left.\begin{aligned} \dot{p} &= v, \\ \dot{v} &= a, \end{aligned}\right\} \tag{2}$$

with a denoting measured acceleration. We ignore measurement noise sources which of course are present.

By *environment* we define the locus E of reflecting surfaces. In order to be able to extract any information on the vehicle's motion, we assume a certain continuity and smoothness, at least in a piecewise sense. Using local linearization the points x on the reflective surfaces satisfy a linear equation $\phi^{\mathrm{T}} x = c$. Assuming that the origin does not belong to any segment, we can write the equation in a normalized form as

$$\phi(x)^{\mathrm{T}} x = 1 \tag{3}$$

which shows the dependance of the normal vector ϕ on the point x.

Assuming a sonar head which continuously scans the environment and a smooth vehicle motion, the point x on the reflecting surface corresponding to the sonar beam is given by $x(t) = p(t) + Ts(t)$ where again p is the vehicle's location vector, s the sonar return and T representing coordinate transformation between the vehicle and the environment reference frames. The matrix T is a function of the vehicle orientation defined by yaw, pitch and roll. In this way we can write a state space equation for the vector ϕ in the time domain, as

$$\begin{aligned} &\dot{\phi}(t) = 0, \quad t \notin \Omega, \\ &\phi(t)^{\mathrm{T}}[p(t) + Ts(t)] + 1, \end{aligned}$$

with Ω denoting the set of times when the slope vector ϕ changes and, with some abuse of notation, $\phi(t) = \phi[p(t) + Ts(t)]$. Clearly at every point of the surface this vector ϕ is orthogonal to the surface, since a perturbation Δx along the surface would yield $\phi^{\mathrm{T}}(x + \Delta x)\phi^{\mathrm{T}} x = 1$ which in turn implies that $\phi^{\mathrm{T}} \Delta x = 0$.

An apparently alternative way of defining the environment would be by a potential function $V(x)$ as discussed by Cristi *et al.* (1995). In this way we define a function $V(x)$ over the whole space, which is such that

(1) $V(x) \geq 0$, for all x,

(2) $V(x) = 0$ if and only if the point x is on the reflecting surface and

(3) for any *small* perturbation Δx the gradient of $V(x)$ is such that $\Delta x' \nabla V(x + \Delta x) \leq 0$, which states that the vector field is attractive towards the reflecting surface, and its gradient is orthogonal to the surface.

The crucial point is property (3) above, which we are going to address. By the very definitions we can see that the two vectors ϕ and ∇V are parallel and describe the local behavior of the surface.

The reason behind the orthogonality of the gradient has been explained by Cristi *et al.* (1995), where we have shown the following; given the dynamic equations

$$\left.\begin{aligned} \dot{p} &= v, \\ \dot{v} &= a, \\ 0 &= V(p + Ts), \end{aligned}\right\} \tag{4}$$

with

$$\nabla V(p + Ts + \tilde{p}) = -\phi\phi^{\mathrm{T}}\tilde{p},$$

the estimator

$$\begin{aligned} \dot{\hat{p}} &= \hat{v} + \kappa \nabla V(\hat{p} + Ts), \\ \dot{\hat{v}} &= a + \mu\kappa \nabla V(\hat{p} + Ts), \end{aligned}$$

is such that

$$p - \hat{p} \to 0, \quad v - \hat{v} \to 0,$$

exponentially fast, as time goes to infinity, provided that the vector field ϕ is *persistently exciting* (*PE*), that is

$$\alpha_1 I \leq \int_t^{t+\alpha_2} \phi(\tau)\phi(\tau)^{\mathrm{T}}\, \mathrm{d}\tau \leq \alpha_3 I$$

for all t and for some constants α_i.

The *proof* has been given by Cristi *et al.* (1995). The meaning of this fact is that, provided that there is sufficient information in the environment (PE of the vector field ϕ), we can estimate the position and velocity of the vehicle by making corrections perpendicular to the surfaces of the reflecting surfaces (the direction of the gradient ∇V). This shows the importance of describing the environment in terms of vectors orthogonal to the surfaces.

The above result shows local convergence of the estimation algorithm, which holds as long as the gradient of the estimator error $\tilde{p}$ is 'small' and we can approximate the gradient of the potential function as $\nabla V(p + Ts + \tilde{p}) = -\phi\phi^{\mathrm{T}}\tilde{p}$. Also the fact that the convergence is exponential guarantees a degree of robustness in the presence of measurement noise and modeling errors.

In the next section we use these models for joint estimation of the vehicle's motion and the environment.

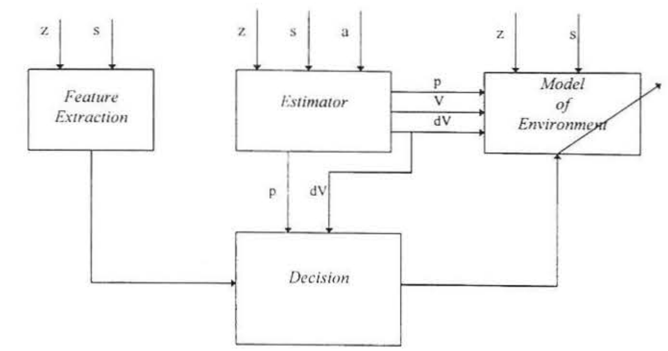

Figure 1. General structure for combined estimation of vehicle motion and model of the environment.

3. Combined motion and model estimation

The ultimate goal of this line of research is to be able to identify the environment in a dynamic fashion, by continuously updating the mathematical model (the potential function V in this case). The ideal mission would be to start with an initial *a priori* estimate of the environment and to refine it during the mission, to include new objects and data collected. An important outcome would be the identification of objects not present in the local map, which could be of importance to the mission.

This recursive approach is in contrast with the batch-processed linear segmentation algorithm proposed by Floyd *et al.* (1991) and Ingold (1992).

The general structure of the sensor-based navigation system is shown in figure 1. The function *feature extraction* is designed to estimate relevent features of the environment, which can be used either to validate or to disprove the mathematical model of the environment. The feature that we have in mind is the slope and orientation of the reflective surface (the vectors ϕ or ∇V) which can be translated into the slope of the corresponding segments in the mathematical model.

A problem with sonar estimation in an underwater environment is its dependence on the motion of the vehicle, whose position and velocity are not directly measurable. In this section we suggest an approach which yields an estimate of the slope of the reflecting surface which is independent of the vehicle's velocity, provided that the acceleration is zero or small. This is the case of a vehicle moving in an environment while current is present.

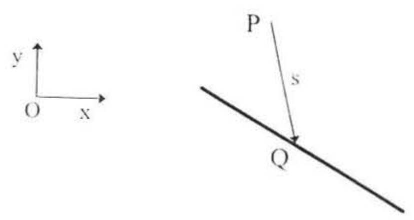

Figure 2. Geometry of slope estimation.

With reference to figure 2, let $p = \mathrm{OP}$ be the location of the sonar head, $q = \mathrm{OQ}$ the location of the reflecting surface, and $s = \mathrm{PQ}$ the sonar beam. Then we can write $q = p + s$ and, by differentiating, we obtain

$$\dot{q}(t) = v(t) + \dot{s}(t), \tag{5}$$

with v being the velocity vector of the vehicle. Using simple calculus, the rightmost term can be computed as

$$\dot{s}(t) = \begin{bmatrix} \cos(\vartheta) & -\rho \sin(\vartheta) \\ \sin(\vartheta) & \rho \cos(\vartheta) \end{bmatrix} \begin{bmatrix} \dot{\rho} \\ \dot{\vartheta} \end{bmatrix}, \tag{6}$$

with ρ and ϑ representing measured sonar range and the sonar heading respectively (the latter in the Earth fixed coordinate frame).

Now consider the case when the sonar takes successive sweeps of the environment by angles $+\alpha$ and $-\alpha$, and denote by the subscripts $+$ and $-$ the corresponding measured vectors. Then we can write (5) for both sweeps, that is

$$\dot{q}_+(t) = v(t) + \dot{s}_+(t),$$

$$\dot{q}_-(t) = v(t) + \dot{s}_-(t),$$

where the velocity vector $v(t)$ is common to both equations. Now the vector $\dot{q}$ represents a vector along the reflecting surfaces, and clearly $\dot{q}_+$ and $\dot{q}_-$ are parallel to each other and most probably of different directions. As a consequence, if we take the difference between the two equations, the velocity term cancels out, which yields

$$\dot{q}_+(t) - \dot{q}_-(t) = \dot{s}_+(t) - \dot{s}_-(t).$$

From this, it is easy to compute the unit vector describing the slope of the surface, as

$$\psi(t) = \frac{\Delta\dot{s}(t)}{\|\Delta\dot{s}(t)\|},$$

where we define

$$\Delta\dot{s}(t) = \dot{s}_+(t) - \dot{s}_-(t)$$

as the difference between the two sonar sweeps. The velocity component perpendicular to the surface can also be computed from (5) as

$$2\mu^{\mathrm{T}} v = \mu^{\mathrm{T}} \dot{s}_+ + \mu^{\mathrm{T}} \dot{s}_-(t),$$

with μ being a unit vector orthogonal to ψ.

This information can be used to validate the current model of the environment for two purposes: firstly to discard measurements which show inconsistencies with the model, and/or secondly to update the mathematical model itself with new segments.

This is implemented by the *decision block* in figure 1, currently under investigation (Cristi *et al.* 1995). Ideally when the model and the measurements are consistent with each other, the following relationships hold:

$$\psi^{\mathrm{T}} \nabla V = 0,$$

$$v - \hat{v} = 0.$$

This approach is currently under investigation. The main problem relies on differentiating the range information in order to compute the two vectors $\dot{s}_+$ and $\dot{s}_-$ above. However, by appropriate low-pass filtering of the signals we obtain the desired smoothness of the sonar range return.

4. On-line updating the model of the environment

In the cases when we attempt to update the map of the environment dynamically, it is important to have a criterion to establish whether a sonar return comes from a mapped or an unmapped object. The framework of Kalman filtering and its interpretation in terms of likelihood is particularly suitable for this class of problems.

By standard Kalman filtering techniques we can determine an estimate of the state $p(t)$, $v(t)$ and its covariance matrix. If we call $P(t)$ the covariance matrix relative to the estimated *position* error, then we can write a likelihood function on the consistency of the sonar return at time t given all past measurements. In particular, assuming that all errors are Gaussian, we can write

$$L(t) = \ln[\Pr\{s(t) \mid s(t-1), \ldots, s(0)\}]$$

$$= C - \tfrac{1}{2} \ln[\nabla V(t)^{\mathrm{T}} P(t) \nabla V(t)]$$

$$- \frac{1}{2} \frac{V(t)^2}{\nabla V(t)^{\mathrm{T}} P(t) \nabla V(t)},$$

with Pr indicating probability, C a constant and the terms V and its gradient ∇V computed using the estimated vehicle position. *This likelihood function is derived from the linear model* (4) *with an extended Kalman filter (EKF) estimator.*

A threshold can be established, mainly by trial and error, by which we can assign the status of *unmapped object* to returns with low probability.

It is easy to see that the likelihood function $L(t)$ is computed recursively from the estimated position of the vehicle. Although by accurate modeling we can assign a suitable threshold with theoretical motivation, it turns out that most of the time some of the modeling assumptions are violated (such as the Gaussian nature of the random signals) and it is rather assigned on a trial-and-error basis. However, we shall see in the applications that the likelihood function has very pronounced local minima corresponding to unmapped targets.

For the joint estimate of the vehicle's motion and the model of the environment we formulate a state-space model which includes the state of the vehicle (position and velocity) and of the environment (orientation of the reflective surfaces).

Combining the vehicle's kinematic model with the environment, and assuming a piecewise constant slope of the reflecting surfaces we can write a joint description of the vehicle's motion and the environment as

$$\dot{p} = v,$$

$$\dot{v} = a,$$

$$\dot{\phi} = 0, \; t \notin \Omega,$$

$$1 = (p + Ts)^{\mathrm{T}} \phi,$$

$$0 = V(p + Ts).$$

The overall system is not completely observable, since if p, ϕ is a solution also $p + \Delta$, $\phi/(1 + \Delta^{\mathrm{T}}\phi)$ is a solution, for any constant Δ. However, assuming that the initial conditions of the vehicle (in terms of the initial position and velocity) are known, we can apply the usual extended Kalman filter (EKF) to estimate the state p, v, ϕ. Changes in the slope ϕ can be detected by an appropriate likelihood function along similar lines as in the previous section.

Using the estimate of the slope ϕ and of the vehicle position p we can determine a matrix of occupancy cells along the lines described by Elfes (1987) and set the potential function within the neighborhood of the estimated surfaces. In particular we set the potential function as

$$V[\hat{p}(t) + Ts(t) + \delta\phi(t)] = f(\delta),$$

$$\nabla V[\hat{p}(t) + Ts(t) + \delta\phi(t)] = g(\delta)\phi(t),$$

with δ a scalar, and $f(\delta)$, $g(\delta)$ scalar functions such that $f(0) = g(0) = 0$. By appropriate discretization we construct the occupancy cells for the environment. In this way we can construct a potential function directly from the estimated location $\hat{p} + Ts$ of the reflecting surface. Using an estimate of the slope ϕ itself we can then construct the function V in the neighborhood of the reflecting surfaces, which satisfies the conditions for V being a potential function.

Suitable likelihood functions have to be determined in order to determine whether or not the potential function needs to be updated. This area is currently under investigation.

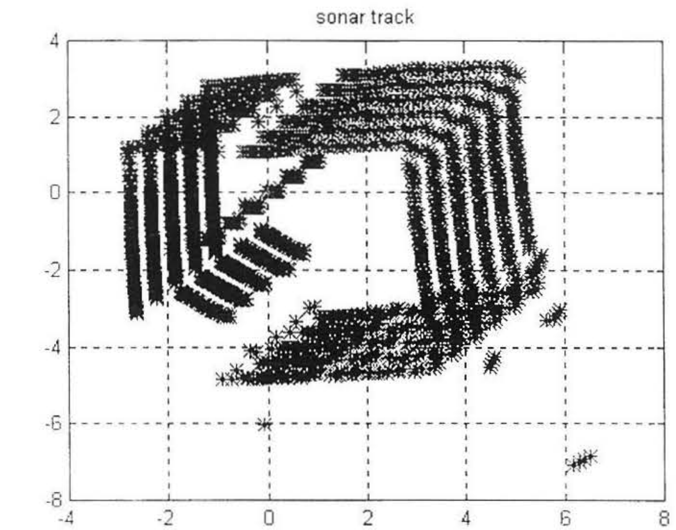

Figure 3. NPS data.

5. Applications and simulations

In this section we describe a number of experiments we performed with the two vehicles (the Naval Postgraduate School's (NPS's) Phoenix and Consiglio Nazionale delle Ricerche-Istituto di Automazione Navale's (CNR-IAN's) Roby). Both vehicles are equipped with a profiling pencil beam sonar head Tritech ST1000 operating at 1.25 MHz, and rotating at increments of 0.9° or 1.8°. Because of the high frequency, under normal operating conditions the range is of the order of 50–100 m. For the experiments the vehicles were operating in a confined environment (test tank for NPS or swimming pool for CNR-IAN). Although this might seem like an artificial setting, problems of reflections and reverberation are actually worse than would be found in an open-ocean environment.

5.1. *Trajectory estimation and detection of unmapped objects*

In the first set of experiments we *assume* the environment to be partially known, in the sense that the vehicle operates on a test tank of known shape and dimensions. What are not known to the vehicle are firstly its initial conditions (position and velocity) with respect to the fixed reference and secondly the presence of two unmarked cylinders. Figure 3 shows the data in Cartesian coordinates, after removing obvious outliers given by unlikely range values.

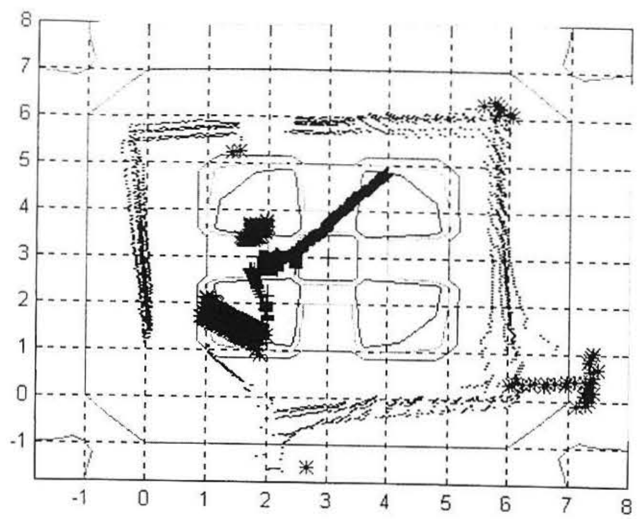

Figure 4. **Trajectory estimation and obstacle detection.**

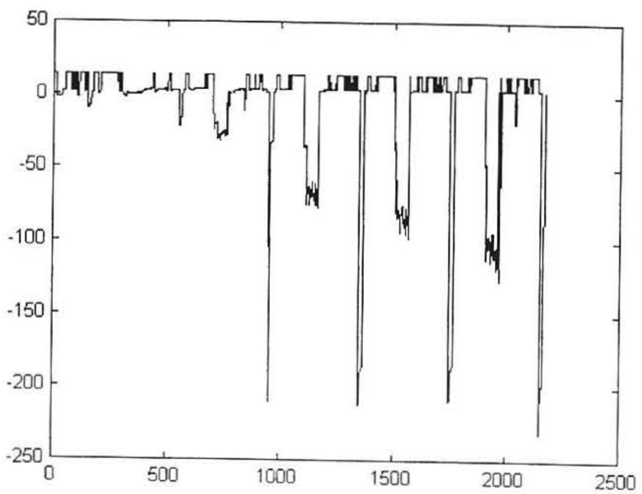

Figure 5. **Likelihood function.**

After the processing described in section 3 with arbitrary initial conditions (center of the tank and zero velocity), the trajectory is estimated and shown as a solid line in figure 4. In this figure we also distinguish between surfaces which are mapped (small dots) and surfaces which are not mapped (asterisks) corresponding to the two cylinders inside the tank and some outliers due to spurious returns of the sonar. The likelihood on which the decision is based is shown in figure 5. Note how the distinction becomes more pronounced (i.e. the picks are larger in magnitude) when the vehicle's location becomes less uncertain, as expected from the Kalman filter formulation.

At this point, it will be the task of a higher level of software to decide how to use this information, which could be either to update the map or just to warn the navigator or the controller.

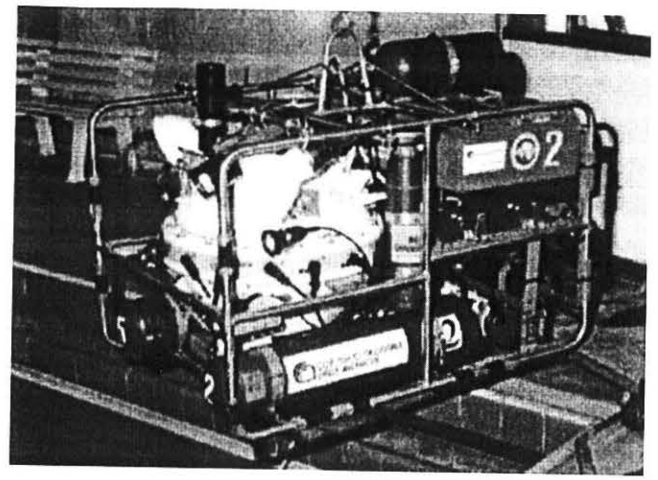

Figure 6. **Tritech ST1000 pencil beam profiling sonar mounted on Roby during Bogliasco (Italy) swimming-pool tests.**

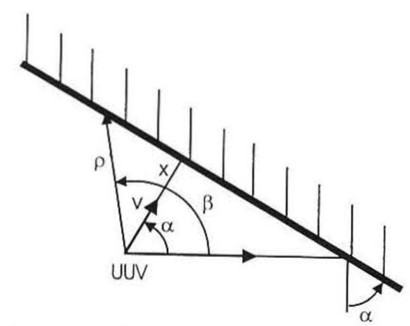

Figure 7. **Geometry for combined trajectory and environment estimation.**

5.2. *Combined trajectory and environment estimation by multiple sweeps*

In this case we assume the environment to be unknown, apart from the fact that it is made of segments which have piecewise constant orientations. For this particular set of problems it is important to determine an appropriate likelihood function to detect transitions between different slopes and suitable criteria for detecting changes in the state space model. In the case of only one surface, the goals are the following: firstly estimation of the orientation in fixed coordinate frame, and secondly estimation of vehicle motion in the component orthogonal to the surface.

The experiments have been conducted by Roby of CNR-IAN (shown in figure 6) in the Bogliasco swimming pool (Olympic size). The algorithm used is a simplified version of what has been presented in the previous sections, and it is based on a three-dimensional state-space formulation (rather than the full six dimensional). With reference to figure 7 the state is given by $[x, v, \alpha]$ = [position, velocity, orientation of

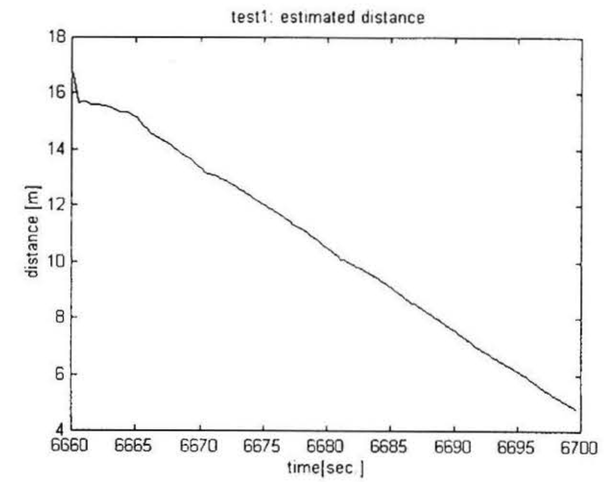

Figure 8.

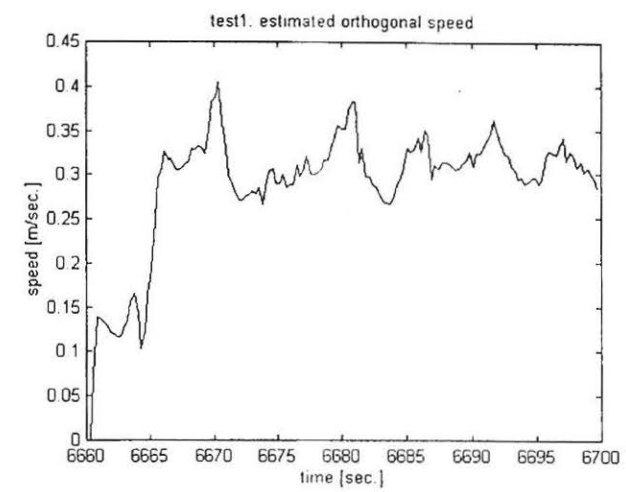

Figure 9.

the surface] and is described by the nonlinear state-space equations

$$
\begin{aligned}
x_1(k+1) &= x_1(k) - x_2(k)\,\Delta t,\\
x_2(k+1) &= x_2(k) + w(k),\\
x_3(k+1) &= x_3(k),\\
y(k) &= \frac{x_1(k)}{\cos[\beta(k) - x_3(k)]} + v(k),
\end{aligned}
$$

from which the EKF updating can be derived by standard arguments.

During the tests the sonar was placed on Roby's longitudinal axis and performed scans measuring between -18 and $+18^\circ$ with 1.8° steps every 0.25 s. The vehicle moved forwards with constant yaw, with a 0.5° maximum error controlled by proportional–integral–differential controller.

Figures 8–11 and figures 12–15 show the results with the vehicle moving towards the wall at a speed of

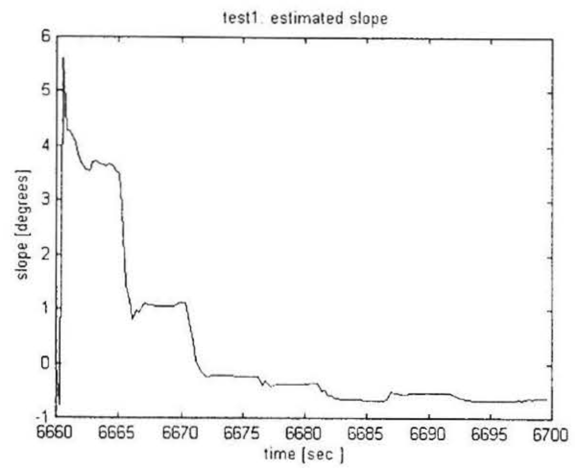

Figure 10.

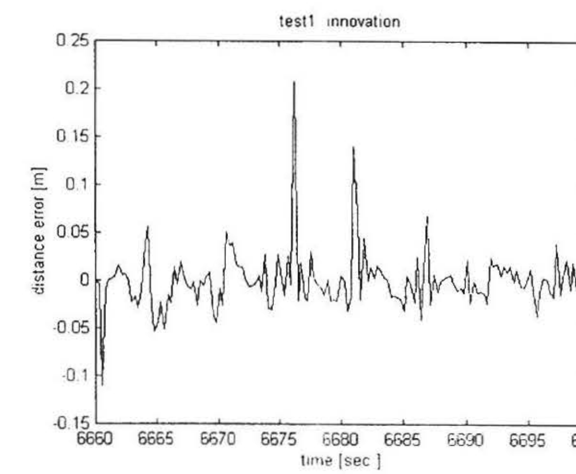

Figure 11.

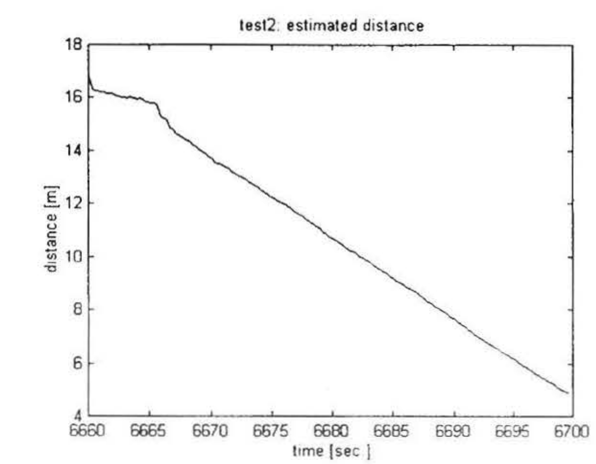

Figure 12.

$0.3\,\mathrm{m\,s^{-1}}$ with two different angles of incidence: 1° in test 1 (figures 8–11), and 7° in test 2 (figures 12–15).

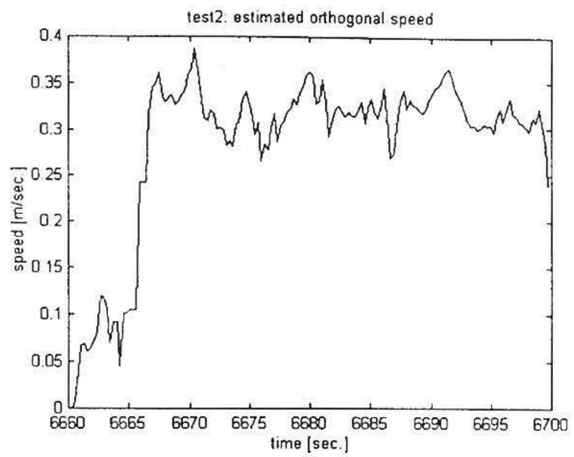

Figure 13.

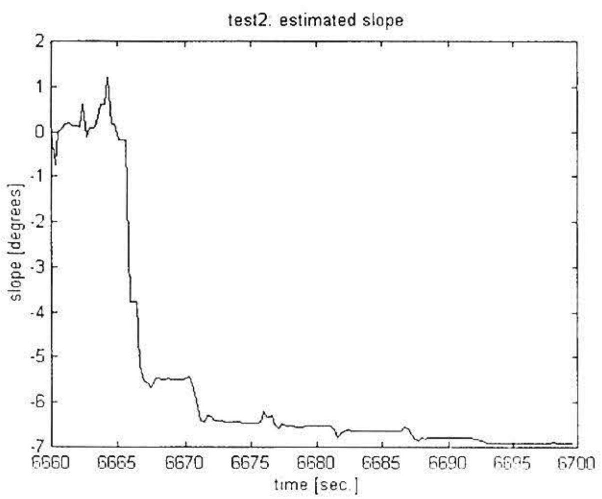

Figure 14.

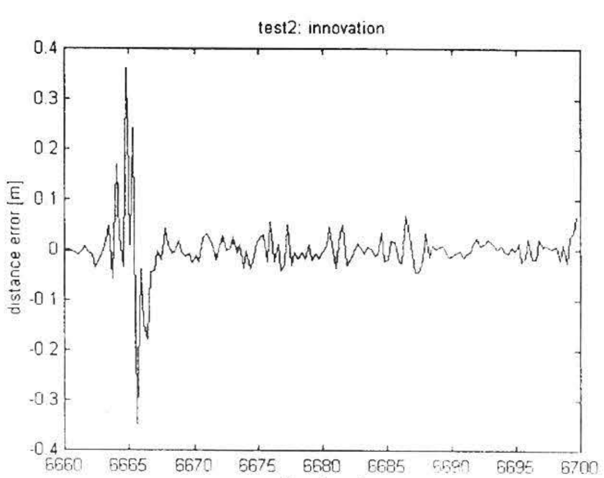

Figure 15.

Measurement noise of the sonar range is estimated to be zero mean with a standard deviation of 0.1. The initial estimates are given by the first measured ping on the wall for position, $0\,\mathrm{m\,s^{-1}}$ for initial velocity and 0° for initial slope estimates.

In particular, figures 8 and 12 show the estimated distance of the vehicle from the wall for the two test cases, while figures 9 and 13 show the estimated velocity component perpendicular to the wall. Both converge to the speed of around $0.3\,\mathrm{m\,s^{-1}}$. The estimates of the inclination of the wall are shown in figures 10 and 14 for the two tests respectively, where the actual values are 1° for test 1, and 7° for test 2. Finally the innovation sequences for each test are given in figures 11 and 15.

From the results we can see the rapid convergence of the estimates of velocity and slope of the wall, particularly when the sonar changes scan direction. Also the estimates stabilize to values consistent with the expectations.

6. Conclusions

The problems of localizing a vehicle within known, partially known or unknown environments have been addressed. They are all formulated on the basis of the same state-space model, which includes the orientation of the reflecting surfaces as part of the state. Several problems are open for research, mainly the choice of a suitable likelihood function to characterize certain events, such as detecting surfaces with different orientations, or objects not accounted in the map. Also real-time implementation issues need to be addressed, within the framework of a hierarchical control structure.

Acknowledgments

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References

Cristi, R., 1993. Sensor based navigation of an autonomous underwater vehicle. *Proceedings of the Eighth International Symposium on Unmanned Untethered Submersible Technology*, Durham, New Hampshire, USA; 1994. Navigation and localization in a partially known environment. *Proceedings of the IEEE 1994 Symposium on Autonomous Underwater Vehicle Technology*, Cambridge, Massachusetts, USA.

Cristi, R., Caccia, M., Veruggio, G., and Healey, A. J., 1995. A sonar based approach to AUV localization. *Proceedings of 3rd IFAC Conference on Control Applications in Marine Systems (CAMS)*, Trondheim, Norway.

Cristi, R., Papoulias, F. A., and Healey, A. J., 1990. Adaptive sliding mode control of autonomous underwater vehicles in the dive plane. *IEEE Journal on Oceanic Engineering*, **15**, 152–160.

Elfes, A., 1987. Sonar based real world mapping and navigation. *IEEE Journal of Robotics and Automation*, **3**, 249–265.

Floyd, C. A., Kanayama, Y., and Magrino, C., 1991. Underwater obstacle recognition using a low-resolution sonar. *Proceedings of the Seventh International Symposium on Unmanned Untethered Submersible Technology*, Durham, New Hampshire, USA.

Gulder, I., and Utkin, V. I., 1995. Sliding mode control for gradient tracking and robot navigation using artificial potential fields. *IEEE Transactions on Robotics and Automation*, **11**, 247–254.

Healey, A. J., Marco, D. B., McGhee, R. B., Brutzman, D. P., and Cristi, R., 1995. A trilevel hybrid control system for the NPS Phoenix autonomous underwater vehicle. *Proceedings of the Joint US–Portugal Workshop in Undersea Robotics and Intelligent Control*, Lisboa, Portugal.

Healey, A. J., Marco, D. B., McGhee, R. B., Brutzman, D. P., Cristi, R., and Papoulias, F. A., 1994a. Coordination of the hovering behaviors of the NPS AUV II using onboard sonar servoing. *Proceedings of the National Science Foundation International Advanced Robotics Programme Workshop on Mobile Undersea Robotics*, MBARI, Pacific Grove, California, USA; 1994b. Tactical execution level coordination for hover control of the NPS AUV II using onboard sonar servoing. *Proceedings of the IEEE 1994 Symposium on Autonomous Underwater Vehicle Technology*, Cambridge, Massachusetts, USA, pp. 129–138.

Ingold, B. J., 1992. AUV navigation from image profile segments using a high frequency sonar. MSME Thesis, Department of Mechanical Engineering, Naval Postgraduate School, Monterey, California.

Moran, B. A., Leonard, J. J., and Chryssostomidis, C., 1993. Geometric shape from sonar ranging. *Proceedings of the Eighth International Symposium on Unmanned Untethered Submersible Technology*, Durham, New Hampshire, USA.

Rimon, E., and Kodistchek, D. E., 1992. Exact robot navigation using artificial potential functions. *IEEE Transactions on Robotics and Automation*, **8**, 501–518.